Physics 1C, Summer Session I, 2011
Final Exam

Instructions: Do any 8 problems. Please do all work on separate sheets of paper, and hand in everything you want to be graded. Clearly mark the problems that you want graded, and box the answers.

Problem 1: A mass on a spring oscillates with an amplitude *A*. What is its displacement from the equilibrium position, in terms of *A*, when the kinetic energy is equal to twice the potential energy?

Solution:
The total energy of the spring is 1/2 kA^2, the potential energy is 1/2 kx^2, and therefore the kinetic energy is 1/2 $k(A^2 - x^2)$. When the kinetic energy is twice the potential energy,

$$\frac{1}{2}k\left(A^2 - x^2\right) = 2\left(\frac{1}{2}kx^2\right)$$

$$A^2 - x^2 = 2x^2 \qquad x^2 = \frac{1}{3}A^2$$

$$\boxed{x = \frac{1}{\sqrt{3}}A}$$

Problem 2: A string with mass per unit length of 0.010 *kg / m* is 0.5 meters long and tied down on both ends. Tension is applied to the string.

(a) What should the tension be to achieve a fundamental frequency of 440*hz*?

(b) If this amount of tension is applied, what is the wave speed in the string?

Solution:
(a) The fundamental frequency of a string with both ends tied down is

$$f_0 = \frac{c}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Solve for the tension:

$$T = 4f_0^2L^2\mu = 4(440s^{-1})^2(0.5m)^2(0.010kg \cdot m^{-1}) = \boxed{1940N}$$

(b) The wave speed is

$$c = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1940N}{0.010kg/m}} = \boxed{440m/s}$$

Problem 3: A dolphin is swimming towards an underwater cliff at a speed of 5 *m/s*. The dolphin emits an ultrasound wave with a frequency of 100 kHz. When the echo returns from the cliff and interferes with the sound from the source, beats are produced. What is the beat frequency? Take the speed of sound in water to be 1500 *m/s*.

Solution:
First find the frequency at the cliff. In this case, we have a moving source (the dolphin) and a stationary observer (the cliff). If f_S is the frequency emitted by the dolphin, then the frequency at the cliff is

$$f_C = f_S \frac{c}{c-v}$$

The cliff reflects the sound at the same frequency. But now the echo is emitted by the cliff (a stationary source) and comes back to the dolphin (a moving observer). The frequency of the echo as heard by the dolphin is

$$f_E = f_C \frac{c+v}{c} = f_S \frac{c+v}{c-v} = 100kHz \frac{1500+5}{1500-5} = 100.669kHz$$

The beat frequency is

$$f_B = f_E - f_S = 0.669kHz = \boxed{669Hz}$$

Problem 4: A red laser (wavelength of 650*nm* in vacuum) is used to send a signal through a fiber-optic cable. The material of the cable has an index of refraction of 1.58.

(a) What is the frequency of the laser, in THz?

(b) What is the wavelength of the laser light in the cable?

(c) If a signal has to travel through 1*km* of cable, how much time does it take between when it is sent and when it arrives?

Solution:
(a) The frequency is

$$f = \frac{c}{\lambda} = \frac{3\times10^8 m/s}{6.50\times10^{-7}m} = 4.61\times10^{14}Hz = \boxed{461THz}$$

(b) The wavelength in the cable decreases by a factor of *n*, so the new wavelength is

$$\lambda = \frac{\lambda_0}{n} = \frac{650nm}{1.58} = \boxed{411nm}$$

(c) The speed of light in the cable is *c* / *n*. Thus the time it takes to travel 1*km* is

$$t = \frac{d}{c/n} = \frac{nd}{c} = \frac{(1.58)(1km)}{3\times10^5 km/s} = 5.27\times10^{-6}s = \boxed{5.27\mu s}$$

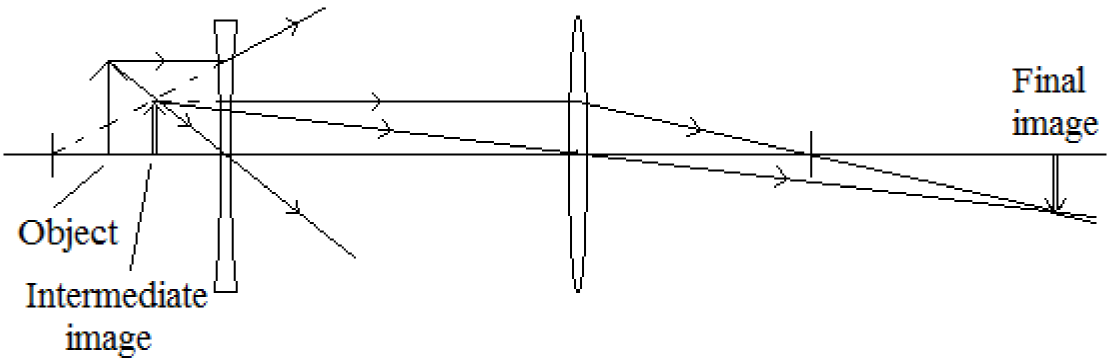
Problem 5: An object is placed 10*cm* in front of a divergent lens with a focal length of -15*cm*. A convergent lens with a focal length of 20*cm* is then placed 30*cm* behind the divergent lens.

(a) Draw a ray diagram, showing where the final image forms.

(b) Calculate the exact location of the final image (give the distance from the convergent lens, and specify if the image is in front or behind this lens).

Solution:

(a)



(b) First find the location of the intermediate image:

$$q_1 = \frac{1}{1/f_1 - 1/p_1} = \frac{1}{1/(-15cm) - 1/10cm} = -6.0cm$$

The intermediate image serves as the object for the second lens. Since the image is 6*cm* in front of the first lens, and the second lens is 30*cm* behind the first lens, the object distance for the second lens is 36*cm*. The location of the final image is therefore

$$q_2 = \frac{1}{1/f_2 - 1/p_2} = \frac{1}{1/20cm - 1/36cm} = \boxed{45cm}$$

The final image forms 45*cm* behind the second lens.

Problem 6: A sound wave with a frequency of 120 *hz* is incident on a wall that reflects sound. The direction of travel of the sound wave is perpendicular to the wall. Since the speed of light in the wall is higher than in air, the wave does not undergo a phase shift upon reflection. At what distances from the wall do the incident wave and the echo interfere constructively? At what distances do they interfere destructively?

Solution:

First determine the wavelength:

$$\lambda = \frac{c}{f} = \frac{330m/s}{120s^{-1}} = 2.75m$$

Let the distance from the wall be *L*. Compared to the sound coming directly from the source, the echo must travel an extra distance of 2*L* (to the wall and back). This is the path length difference. For constructive interference,

$$2L = j\lambda \qquad L = \frac{1}{2}j\lambda = \boxed{(1.375m)j = 0m,\ 1.375m,\ 2.75m,\ ...}$$

For destructive interference,

$$2L = \left(j + \frac{1}{2}\right)\lambda \quad L = \left(j + \frac{1}{2}\right)\frac{1}{2}\lambda = \boxed{\left(j + \frac{1}{2}\right)(1.375m) = 0.69m,\ 2.06m,\ 3.44m,\ ...}$$

Problem 7: A spaceship travels to a star 200 light-years away. If the trip takes only 30 years according to the ship's crew, how fast is the spaceship moving (as a fraction of the speed of light)? Neglect the time it takes for the ship to accelerate to that speed.

Solution:
The amount of time the trip takes according to an observer on Earth is $t = d / v$, where d is the distance and v is the speed of the ship. The time t is greater than the time measured by an observer on board the spaceship by a factor of γ. Thus $t = \gamma t_0$. Therefore,

$$\gamma t_0 = \frac{d}{v} \qquad \gamma v = \frac{v}{\sqrt{1 - v^2/c^2}} = \frac{d}{t_0} \qquad v^2 = \left(1 - \frac{v^2}{c^2}\right)\frac{d^2}{t_0^2}$$

$$\frac{v^2}{c^2}\left(1 + \frac{d^2}{c^2 t_0^2}\right) = \frac{d^2}{c^2 t_0^2}$$

$$\frac{v}{c} = \frac{d/(ct_0)}{\sqrt{1 + d^2/(ct_0^2)}} \qquad \frac{d}{ct_0} = \frac{200ly}{(1ly/yr)(30yr)} = 6.67$$

$$\frac{v}{c} = \frac{6.67}{\sqrt{1 + 6.67^2}} = \boxed{0.989}$$

Problem 8: A proton has a rest energy of $m_P c^2$ = 938MeV. If the proton is to be accelerated to 95% of the speed of light, what voltage must it be accelerated through?

Use the relationship between kinetic energy and speed:

$$E = (\gamma - 1)\,mc^2 = \left(\frac{1}{\sqrt{1 - 0.95^2}} - 1\right) \times 938MeV = 2066MeV$$

The proton has the same charge as an electron (up to a sign), so like the electron it must be accelerated through 1V for every eV of energy gained. Thus to achieve a speed of 0.95*c*, the proton must be accelerated through **2.066 GV**.

Problem 9: A galaxy is observed through a telescope. A spectral line that has a wavelength of 420*nm* when the source is at rest is found at a wavelength of 600*nm*. Assume that the galaxy is moving either directly towards Earth or directly away from it, not across the field of view. Is it moving towards or away? What is its speed, as a fraction of the speed of light?

Solution:
The wavelength of the source increases, so the frequency decreases. Thus the galaxy is moving away from the Earth.

Solve the equation for the Doppler effect for the speed:

$$f = f_0\sqrt{\frac{1-\beta}{1+\beta}} \qquad \sqrt{\frac{1-\beta}{1+\beta}} = \frac{f}{f_0} = \frac{\lambda_0}{\lambda}$$

$$1-\beta = \frac{\lambda_0^2}{\lambda^2}(1+\beta) \qquad \left(1+\frac{\lambda_0^2}{\lambda^2}\right)\beta = 1-\frac{\lambda_0^2}{\lambda^2}$$

$$\beta = \frac{1-\lambda_0^2/\lambda^2}{1+\lambda_0^2/\lambda^2} = \frac{1-(420/600)^2}{1+(420/600)^2} = 0.342$$

$$\boxed{v = 0.342c}$$

Problem 10: Tritium is an isotope of hydrogen consisting of one proton and two neutrons. The tritium nucleus has a rest energy of m_Tc^2 = 2808.8182 MeV. It undergoes beta decay, turning into helium 3 (two protons and one neutron) and emitting an electron and an electron antineutrino. The helium 3 nucleus has a rest energy of $m_{He3}c^2$ = 2808.2887 MeV, the electron has a rest energy of m_ec^2 = 0.5110 MeV, and the antineutrino has a rest energy that is very close to zero.

(a) What is the maximum kinetic energy that the emitted electron could have, in keV?

(b) How fast would the electron emitted with maximum possible kinetic energy be moving, as a fraction of the speed of light?

(c) Is it possible to emit the electron with zero kinetic energy? Explain.

Solution:

(a) The energy can be split in any way between the electron and the antineutrino, so the electron could end up with all the kinetic energy. By conservation of energy, the kinetic energy released is the amount by which the total rest energy decreases:

$$E_K = (2808.8182 - 2808.2887 - 0.5110)MeV = 0.0185MeV = \boxed{18.5keV}$$

If all this happened to go to the electron, the electron would have a maximum kinetic energy of 18.5 keV.

(b) This energy is actually low enough compared to the electron's rest energy of 511keV that a non-relativistic approximation would give a fairly accurate result, but let us use the relativistic kinetic energy anyway. Write down the expression for the kinetic energy and solve for *v*:

$$E_K = (\gamma - 1)\, mc^2$$

$$\gamma = 1 + \frac{E_K}{mc^2} = 1 + \frac{18.5keV}{511keV} = 1.036$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad 1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2} \qquad \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{1.036^2}} = 0.261$$

$$\boxed{v = 0.261c}$$

(c) Yes, it is possible. All the kinetic energy could go to the antineutrino. Then the electron would be produced at rest with respect to the nucleus.